Extrapolative Expectations and the Equity Premium

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Abstract

Many stockholders irrationally believe that high recent market returns predict high future market returns. I argue that the presence of these extrapolative investors can help resolve the equity premium puzzle if the elasticity of intertemporal substitution (EIS) is greater than unity. Extrapolators' overreaction to dividend news generates countercyclical expected returns. Rational investors respond by making their consumption growth more procyclical. The equity premium is high because extrapolators believe stocks are a bad hedge and rational investors have high consumption growth covariance with stocks. I match the U.S. data with a relative risk aversion of 4 and an EIS of 2.

JEL classification: D51, E44, G12

Keywords: equity premium puzzle, riskfree rate puzzle, equity volatility puzzle, noise traders, irrational expectations, adaptive expectations, heterogeneous agents, elasticity of intertemporal substitution

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1 Introduction

Models seeking to explain the high, time-varying equity premium must contend with the growing body of evidence that the typical investor does not seem to have rational expectations about the premium. Survey and experimental studies show that the mean subjective forecast of the aggregate stock market return is positively correlated with recent returns (Andreassen and Kraus (1990), De Bondt (1993), Durell (2001), Fisher and Statman (2000), Qiu and Welch (2004), Vissing-Jørgensen (2003)), even though actual market returns exhibit no positive serial correlation (Fama and French (1988b)).[1] In fact, Durell (2001) finds that average investor optimism about the stock market *negatively* predicts future returns. Further evidence that these extrapolative beliefs are mistaken comes from the return forecasts of more sophisticated market observers—such as professional economists, institutional investors, and investment newsletter editors—which are contrarian (De Bondt (1991), Shiller (2000), Clarke and Statman (1998)).

Even if extrapolative beliefs were present only among poor to moderately wealthy stockholders, they would play an important role in the response of aggregate consumption to stock returns because these households' share of aggregate consumption is large. The bottom two stockholder consumption deciles accounted for 10.6% of total stockholder nondurable and services consumption in 1998. The next three deciles accounted for 23.3%.[2]

However, the incidence of extrapolative beliefs does *not* diminish swiftly with wealth. Qiu and Welch (2004) report a 97% correlation between the returns expectations of the wealthy and poor. Graham and Harvey (2001) find that U.S. chief financial officers have extrapolative returns forecasts. Therefore, extrapolative beliefs are likely to have a significant price impact as well.

This paper argues that the presence of return extrapolators can help resolve the equity premium puzzle. I present a general equilibrium model of an endowment economy where aggregate consumption growth has low covariance with market returns and relative risk aversion is low. Nonetheless, the average equity premium is high. Furthermore, stock returns are much more volatile than dividend growth and do not predict future dividend growth,

[1] It is true that there is a small amount of postive serial correlation in index returns over a horizon of a few months (Lo and MacKinlay (1988)). However, investor expectation surveys typically ask about longer horizons, where return autocorrelations are negative.

[2] I use the classification of stockholders used by Vissing-Jørgensen (2002) to compute nondurable and services consumption from the Consumer Expenditure Survey, where observations are dropped for households reporting wages less than the statutory minimum wage. Nondurable and services consumption is the ND measure used in Krueger and Perri (2005).

which implies that stock returns are mean-reverting.[3] The riskfree rate is low, stable, and does not predict aggregate consumption growth. I find that I can match the postwar U.S. equity Sharpe ratio and mean riskfree rate with a relative risk aversion coefficient of 4 and an elasticity of intertemporal substitution (EIS) of 2 when extrapolators constitute 75% of the population.

Extrapolators generate an anomalously high equity premium through three channels. First, they make stocks highly risky, even though stocks are claims to a fundamentally low-risk technology. Extrapolators are more willing to buy stocks when there has been recent good news, and they are less willing to buy stocks when there has been recent bad news. In equilibrium, this generates procyclical price-dividend ratios, which lead to countercyclical expected stock returns because dividend growth is independently and identically distributed over time. Sophisticated investors with rational expectations respond by making their consumption more procyclical, since an EIS greater than 1 causes lower expected returns to increase consumption today. Sophisticates' consumption growth therefore has a high covariance with equity returns, leading sophisticates to demand a high average equity return. This is consistent with Aït-Sahalia, Parker, and Yogo's (2003) finding that the consumption of the very rich, who are likely to be disproportionately (although not universally) rational, can be better reconciled with the equity premium.

Second, extrapolators believe stocks to be a poor hedge, since they think that low stock returns today predict low future stock returns. Therefore, they too demand a high equity premium on average. In my model, extrapolators' unconditional expectation of the equity premium is correct. It is only their *conditional* expectations that are incorrect.

Third, extrapolators obscure the risk they create in stocks. Extrapolators' returns expectations move in the opposite direction of sophisticates' contrarian expectations. Therefore, when sophisticates decrease their savings rate in response to a positive innovation in stock prices, extrapolators increase their savings rate because they think investment prospects have improved. Choi et al. (2004) find empirically that 401(k) investors cut their consumption growth in response to positive capital gains. Aggregate consumption statistics add together sophisticate and extrapolator consumption, creating an aggregate series whose growth has low covariance with stock returns.

[3] Shiller (1981) and LeRoy and Porter (1981) document excess volatility in stock returns relative to dividend growth. Cochrane (1992) finds stock returns do not predict future dividend growth. Cochrane (1991), Campbell and Shiller (1988a, 1988b), Fama and French (1988a), and Hodrick (1992) document mean reversion in stock returns.

The riskfree rate is low because sophisticates' consumption growth is very volatile, generating a strong precautionary savings motive, while extrapolators are irrationally afraid that stocks are bad hedges. The riskfree rate is stable because the procyclicality of extrapolator demand for the riskfree asset is offset by the countercyclicality of sophisticate demand. Furthermore, both extrapolator and sophisticate demands are individually stable because their high EIS implies a weak consumption smoothing motive.

The model's results rest crucially on the assumption that the EIS is greater than unity. Although the empirical literature has not come to a consensus on the size of the EIS, a number of studies using micro data have found the EIS to be large. Gourinchas and Parker (2002) and Gruber (2006) estimate an EIS of 2, Attanasio, Banks, and Tanner (2002) estimate an EIS of 1.54, and Vissing-Jørgensen and Attanasio (2003) obtain estimates between 1.03 and 2.34.

It is commonly argued that even if individuals' beliefs are irrational, their choices are rational because they are delegated to financial institutions. However, it is doubtful that financial institutions completely neutralize the effect of extrapolative beliefs because institutions' role is typically restricted to allocating money within an asset class. They usually do not control the household's consumption-savings decision and the fraction of household wealth allocated to each asset class.[4] If extrapolators decide to invest more in equities, it is the money manager's job to allocate that additional money optimally among stocks, even if she thinks that equities overall are overvalued. The model's results hold even if every stock is correctly valued relative to every other stock.

Previous theoretical work has also shown that the presence of irrational noise traders can affect stock prices despite the presence of rational arbitrageurs (De Long et al. (1990), Campbell and Kyle (1993), Shleifer and Vishny (1997), Barberis and Shleifer (2003)). Empirical studies that find important limits to the extent arbitrageurs can move prices towards fundamental values include Lee, Shleifer, and Thaler (1991), Froot and Dabora (1999), Mitchell, Pulvino, and Stafford (2002), Wurgler and Zhuravskaya (2002), and Lamont and Thaler (2003). To date, however, there has been little exploration of the effect noise traders have on the joint behavior of consumption and asset returns.[5]

[4]Financial institutions may not be given such a role because they cannot credibly communicate optimal savings rates and asset allocations when their compensation is based on total assets under management and varies by asset class.

[5]An exception is Ingram (1990), who models an economy populated by sophisticates and noise traders who follow rule-of-thumb consumption and portfolio rules. She obtains an equity premium and volatility that are both an order of magnitude below the values observed in the data. De Long et al. (1990) suggest,

Section 2 presents the model. Section 3 describes the parameter values used to generate the results in Section 4. Section 5 concludes. An appendix describes the computational procedure used to numerically solve the model.

2 Model description

Following Lucas (1978) and Mehra and Prescott (1985), I consider an infinite-horizon endowment economy where there is one share of equity that pays the aggregate consumption endowment as a stochastic perishable dividend $\bar{C}_t$ each period. There is also a riskfree asset present in zero net supply. The ex-dividend price of the risky asset at time t is P_t, and the riskfree rate is $R_{f,t+1}$.

There are two types of finitely-lived agents: extrapolators, present in measure Q, and sophisticates, present in measure $1-Q$. Both types have recursive utility of the form derived by Epstein and Zin (1989) and Weil (1989),

$$U_t = \left[(1-\beta) C_t^{(1-\gamma)/\theta} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\theta} \right]^{\theta/(1-\gamma)}, \tag{1}$$

where C_t is the investor's consumption at time t, γ is the coefficient of relative risk aversion, ψ is the EIS, $\theta \equiv (1-\gamma)/(1-1/\psi)$, and $\beta < 1$ is the time discount factor.[6] Investors have no bequest motive.[7]

Log dividend growth is independently and identically normally distributed with mean $\bar{g}$ and variance σ_g^2. Sophisticates know the true dividend growth process, but extrapolators believe next period's log dividend growth, $\log(\bar{C}_{t+1}/\bar{C}_t)$, is normally distributed with mean $\hat{g}_t$ and variance σ_g^2, where $\hat{g}_t$ is the geometrically weighted mean of present and past log dividend growth realizations:

$$\hat{g}_t = (1-\phi) \sum_{\tau=0}^{\infty} \phi_{t-\tau}^{\tau} \log\left(\bar{C}_{t-\tau}/\bar{C}_{t-\tau-1} \right), 0 < \phi < 1. \tag{2}$$

as I do, that the riskiness of stocks may be both caused and hidden by noise trader consumption. However, they do not formally model this effect.

[6] When $\gamma = 1/\psi$, Epstein-Zin-Weil utility is equivalent to constant relative risk aversion utility.

[7] Hurd (1989) and Gan et al. (2004) find that nearly all bequests in the U.S. are accidental. In the absence of a bequest motive, there would be a strong desire among investors to annuitize their wealth (Yaari (1965)). Annuitization is exogenously precluded in the model. Empirically, very few people hold private annuities. This outcome could be achieved endogenously by giving individuals private information about their mortality risk, causing adverse selection to dry up the annuity market.

This inference is rational if the expected log dividend growth rate is a random walk. (See Barsky and De Long (1993)).

Because the support of the log dividend growth distribution is infinite, investors are unwilling to leverage or short the risky asset for fear of ending up with negative wealth and infinite marginal utility. The risky asset is the only asset in positive net supply, so all investors invest their entire portfolio in the risky asset. This restriction considerably simplifies the calculation of equilibrium because the risky asset portfolio share policy function need not be solved for and asset prices do not affect wealth dynamics except through their effect on agents' consumption behavior.[8]

The timing of the model is as follows. At the beginning of each period, securities pay their dividend. Securities are then traded and consumption occurs. At the end of the period, a fixed positive fraction $\delta \leq 1 - \beta$ of the population is stochastically chosen to die. New investors are born to replace deceased investors, whose assets are evenly distributed among the newborns.[9] The number of newborns each period is fixed to maintain a constant total population, and the fraction of extrapolators among newborns is Q.

Let ω_t be the total number of risky asset shares held by extrapolators at the beginning of period t, and $\hat{C}_t/\hat{W}_t$ be their consumption-wealth ratio. To clear the asset market, sophisticates hold $1 - \omega_t$ shares. After consumption and trading, but before mortality and redistribution, extrapolators hold $\tilde{\omega}_t$ shares, where

$$\tilde{\omega}_t = \left(\frac{\bar{C}_t}{P_t} + 1\right)\left(1 - \frac{\hat{C}_t}{\hat{W}_t}\right)\omega_t \tag{3}$$

The right side of equation (3) is the extrapolators' wealth at the beginning of the period times the savings rate divided by the price of an equity share.

After mortality and redistribution, the number of shares extrapolators hold going into period $t + 1$ is

$$\omega_{t+1} = (1 - \delta)\,\tilde{\omega}_t + \delta Q. \tag{4}$$

The first term on the right side of this equation is the number of shares held by extrapolators

[8] If there are multiple traded assets, next period's wealth distribution depends not only on investors' consumption and portfolio choices, but also on assets' return realizations. However, next period's asset prices will be a function of next period's wealth distribution. Therefore, a fixed-point problem must be solved in order to compute wealth dynamics.

[9] I assume that death occurs suddenly, so that there is no opportunity for an investor to liquidate all of his assets for consumption upon learning that death is imminent. The distribution rule is equivalent to assuming that a sophisticate's heir is not disproportionately likely to be sophisticated him or herself.

who survive into $t+1$. The second term is the total number of shares that are bequested multiplied by the proportion of those shares that go to extrapolators.

Equities are priced to satisfy both investor types' Euler equations. Let C_t^* be aggregate sophisticate consumption, $\hat{C}_t$ be aggregate extrapolator consumption, and $\hat{E}$ represent the expectations operator under the extrapolators' probability measure. The Euler equations are

$$E_t\left[\beta^\theta\left(\frac{1-\tilde{\omega}_t}{1-\omega_{t+1}}\cdot\frac{C_{t+1}^*}{C_t^*}\right)^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^\theta\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^\theta\right] = 1 \tag{5}$$

$$\hat{E}_t\left[\beta^\theta\left(\frac{\tilde{\omega}_t}{\omega_{t+1}}\cdot\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^\theta\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^\theta\right] = 1, \tag{6}$$

subject to the consumption market clearing condition $\bar{C}_t = C_t^* + \hat{C}_t$ for all t.

Notice that the consumption growth in the Euler equations does not equal the aggregate consumption growth of the investor's type. This is because of mortality and redistribution; simply using aggregate consumption growth of the investor type would include the consumption of investors not alive when assets are being traded in time t. Instead, the relevant consumption growth is that of investors who are alive at the beginning of both t and $t+1$. The homotheticity of the Epstein-Zin-Weil utility function implies that surviving sophisticates' share of aggregate sophisticate consumption each period equals their share of aggregate sophisticate wealth. Survivors hold $(1-\delta)(1-\tilde{\omega}_t)$ equity shares coming into period $t+1$, whereas on aggregate sophisticates hold $1-\omega_{t+1}$ shares coming into period $t+1$. Coming into period t, sophisticates who survive into period $t+1$ hold $(1-\delta)(1-\omega_t)$ shares, whereas aggregate sophisticate shares coming into period t is $1-\omega_t$. Hence, aggregate sophisticate growth must be multiplied by $(1-\tilde{\omega}_t)/(1-\omega_{t+1})$ in the Euler equation. Analogous reasoning applies for extrapolator consumption growth.

Because the riskfree asset is not traded, the riskfree rate is the shadow riskfree rate, determined by the minimum riskfree rate (i.e. highest riskfree asset price) any investor is willing to accept. Let $R_{f,t+1}^*$ be the sophisticates' shadow riskfree rate and $\hat{R}_{f,t+1}$ be the extrapolators' shadow riskfree rate. The Euler equations for the riskfree rate are

$$
\begin{aligned}
1 &= E_t\left[\beta\left(\frac{1-\tilde{\omega}_t}{1-\omega_{t+1}}\cdot\frac{C^*_{t+1}}{C^*_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^{\theta-1}\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\theta-1}\right]R^*_{f,t+1} && (7)\\
1 &= \hat{E}_t\left[\beta\left(\frac{\tilde{\omega}_{t+1}}{\omega_{t+1}}\cdot\frac{C^*_{t+1}}{C^*_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{P_{t+1}/\bar{C}_{t+1}+1}{P_t/\bar{C}_t}\right)^{\theta-1}\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\theta-1}\right]\hat{R}_{f,t+1}. && (8)
\end{aligned}
$$

The economy's shadow riskfree rate is then

$$R_{f,t+1}=\min\{R^*_{f,t+1},\hat{R}_{f,t+1}\}. \tag{9}$$

I solve for equilibrium numerically. In order to do so, I discretize the dividend growth process using the Tauchen and Hussey (1991) approximation to a normally distributed process based on a Gauss-Hermite quadrature rule. The approximation can be described by the set of ordered pairs, $\{(g_k,\pi_k)\}_{k=1}^{K}$, where π_k is the probability that the log growth realization is g_k, and $\sum_{k=1}^{K}\pi_k=1$. I set $K=9$.

I now construct K mappings $\{\hat{\pi}_j:[g_1,g_K]\to\mathbb{R}\}_{j=1}^{K}$ between $\hat{g}$, extrapolators' expectations of mean log dividend growth, and their beliefs about the probability that π_j will be the log growth realization next period. The goal is to create a discrete approximation of the belief that log dividend growth is distributed $N\left(\hat{g}_t,\sigma_g^2\right)$ while holding fixed the discrete set of possible dividend growth realizations.

I begin by defining $\hat{\pi}_j$ over the domain $\{g_1,g_2,...,g_K\}$ (that is, when $\hat{g}$ is equal to one of the possible discrete realizations of log dividend growth). Define the mappings $\left\{\xi_j:\{g_1,g_2,...,g_K\}\to\mathbb{R}\right\}_{j=1}^{K}$ as

$$\xi_j\left(g_k\right)=\begin{cases}\pi_{j-(k-\lceil K/2\rceil)} & \text{if } k-\lceil K/2\rceil<j<k+\lceil K/2\rceil\\ 0 & \text{otherwise}\end{cases}. \tag{10}$$

Then

$$\hat{\pi}_j\left(g_k\right)=\frac{\xi_j\left(g_k\right)}{\sum_{i=1}^{K}\xi_i\left(g_i\right)}. \tag{11}$$

In words, what I have done is give extrapolators the correct probability beliefs when $\hat{g}_t$ is equal to $g_{\lceil K/2\rceil}$, which is the unconditional mean of $\hat{g}$ when K is odd as I have stipulated. When $\hat{g}_t$ equals, say, $g_{\lceil K/2\rceil+2}$, I shift the probability mass that is on $g_{\lceil K/2\rceil}$ under the true probability measure to $g_{\lceil K/2\rceil+2}$, the probability mass on $g_{\lceil K/2\rceil-1}$ under the true measure to $g_{\lceil K/2\rceil+1}$, and so on. Because the true probability masses on g_{K-1} and g_K run "off the grid" under this shifting algorithm, I eliminate their mass and renormalize the remaining probability masses so they sum to 1.

I then define values of $\hat{\pi}_j$ when $\hat{g} \notin \{g_1, g_2, ..., g_K\}$ using cubic spline interpolation with "not-a-knot" end conditions, where the spline nodes are $\{g_1, g_2, ..., g_K\}$ and the values at those nodes are $\{\hat{\pi}_j(g_1), \hat{\pi}_j(g_2), ..., \hat{\pi}_j(g_K)\}$.[10]

Figure 1 shows how well this mapping succeeds at making the expected log dividend growth equal to $\hat{g}_t$ and the expected standard deviation constant at σ_g. The top panel plots the extrapolators' expected log dividend growth against $\hat{g}$. The bottom panel plots extrapolators' belief about the standard deviation of log dividend growth against $\hat{g}$. A perfect approximation would lie on the gray 45-degree line in the top panel and on the gray horizontal line in the bottom panel. The figure demonstrates that the mapping comes extremely close to this benchmark.

I solve for a recursive equilibrium defined over the state space $S \equiv (\omega, \hat{g})$ using the time-iteration algorithm described in Judd, Kubler, and Schmedders (2000). There are two functions to solve for: the price-dividend ratio and the extrapolator consumption-wealth ratio. Details of the computational procedure are in the Appendix.

3 Model calibration

There are eight parameters that must be chosen for the model, which is simulated at a quarterly frequency. These parameters are summarized in Table 1, where I have annualized numbers when appropriate. I choose the mean of log consumption dividend growth, $\bar{g}$, to match the U.S. postwar aggregate per capita mean for nondurables and services. The annual standard deviation of aggregate postwar per capita consumption growth is 1.07%, but empirical studies have found that stockholder consumption is significantly more volatile than nonstockholder consumption. This paper is concerned with stockholder behavior, since one does not expect the Euler equation for equities to hold for agents whose portfolios are at a corner solution. Therefore, I wish to choose consumption dividend volatility to match stockholder consumption volatility.[11] I use the midpoint of the Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002) estimates for $2\sigma_g$.

[10] See de Boor (2001) for an introduction to the theory of splines.

[11] Mimicking the more volatile stockholder consumption series makes the equity premium puzzle easier to resolve because the puzzle is fundamentally about the smoothness of consumption growth.

The parameter ϕ, which is the autocorrelation of $\hat{G}$, is chosen to match the first-order autocorrelation of the postwar price-dividend ratio because I find that the simulated price-dividend ratio autocorrelation is usually very close to ϕ. I set the quarterly death rate δ so that each agent has an expected investing lifetime of $1/4\delta = 50$ years, roughly corresponding to the number of years a U.S. resident lives after reaching age 21. There is little guidance from the empirical literature on the exact proportion of extrapolators in the population. However, the evidence is strong that extrapolators outnumber contrarians. I therefore set Q, the proportion of extrapolators in the stockholding population, to 0.75.

Finally, I choose the preference parameters. I fix the EIS ψ at 2, which is the point estimate of Gourinchas and Parker (2002) and Gruber (2005). I then vary risk aversion γ and time discount factor β as free parameters until the model's results match the data. I find that a γ of 4 and an annualized β of 0.974 produce the desired results.

4 Results

4.1 Asset returns and prices

I draw a 207-quarter period of the economy (corresponding to the length of the post-war period reported in Campbell (2003)) by starting the system at $\omega = Q$ and $\hat{g} = \bar{g}$, simulating a 1,207-quarter sample, and then discarding the first 1,000 quarters of data. I repeat this procedure 1,000 times in order to obtain a distribution of return paths.

Table 2 shows the model's simulated moments, as well as the corresponding moments when there are only sophisticates in the economy. The sophisticate-only case demonstrates the classic equity premium and riskfree rate puzzles: the equity premium and volatility are much too low and the riskfree rate is much too high. On the other hand, the model with extrapolators can closely match the data's 0.90% mean log riskfree rate in its median simulation run with reasonable preference parameters. In addition, the riskfree rate is very stable. Campbell (2003) observes that the true riskfree rate must be more stable than the real riskfree rate calculated from the data, since the latter includes *ex post* inflation shocks. 90% of the simulation runs yield riskfree rate standard deviations between 0.09% and 0.34%, a range considerably below the 1.75% standard deviation observed in the postwar period.

The model's median equity Sharpe ratio is close to that in the data. The corresponding equity premium and volatility are high but a little below the moments of the postwar data. This is to be expected, since equity in the model is a claim to the entire economy's consumption, whereas in reality, equity is a leveraged claim to consumption. Between 1945 and 1998, debt as a percent of total firm market value averaged 35.8% for U.S. nonfarm nonfinancial corporations, according to the Flow of Funds Accounts. To see how much leverage is required for the model's median result to match the data, I start with the formula

$$R_{l,t+1} = \frac{(1 + R_{t+1}) - (1 + R_{f,t+1})\, D_t/V_t}{1 - D_t/V_t}, \tag{12}$$

where $R_{l,t+1}$ is the levered return, V_t is the value of the entire firm, and D_t is the market value of debt that pays the riskfree rate.[12] I assume the firm maintains a constant debt-to-value ratio through debt-for-equity or equity-for-debt swaps that all shareholders participate in proportionally at fair market value. I ignore general equilibrium price effects that may arise from carving the claim to all corporate cash flows into levered equity and corporate debt securities.

I can then take unconditional expectations of both sides of (12) and rearrange to obtain the following expression for the leverage required to reconcile the observed levered return with the model's unlevered return:

$$\frac{D}{V} = \frac{E\left(R_{l,t+1}\right) - E\left(R_{t+1}\right)}{E\left(R_{l,t+1}\right) - E\left(R_{f,t+1}\right)}. \tag{13}$$

The average quarterly real arithmetic return on the market in the data is 2.02%. The mean arithmetic return in the model's median simulation run is 1.16% for the risky asset and 0.24% for the riskfree asset. This yields a required debt-to-value ratio of 48.9% to appropriately scale up the model's equity return and volatility, which is somewhat higher than the historical average.

Recall that I have set Q, the proportion of extrapolators in the economy, to 0.75. Because they hold incorrect conditional beliefs, extrapolators are on average poorer than sophisticates. In the median simulation run, extrapolators hold 47.9% of the economy's wealth on average, and there is not much variation in this figure across simulation runs. In the 1st percentile run, extrapolators hold 46.1% of the wealth on average, and in the 99th percentile run, extrapolators hold 50.1% of the wealth. The median difference between the maximum and minimum extrapolator wealth share within a run is only 1.8 percentage points.

[12] The formula ignores the convexity induced by equity's limited liability, since it is implausible that the entire corporate equity market would lose all its value. By a similar token, the minimal chance of aggregate corporate bankruptcy causes the corporate bond to pay the riskfree rate.

Because it is difficult to visualize three-dimensional surfaces when drawn on a two-dimensional page, and because the extrapolator wealth share is so stable, I will graph the model solutions at the cross-section of the state space where extrapolators have 50% of the wealth. Figure 2 graphs the shadow riskfree rates of both investor types at this cross-section. The prevailing riskfree rate is the lower envelope of these two series. The distance between adjacent data markers in a series represents a one-standard-deviation movement in $\hat{g}$, with the middle marker denoting the solution when $\hat{g} = \bar{g}$, its mean.

There are three things to note in this graph. The first is that extrapolator and sophisticate riskfree asset demand move in opposite directions with respect to recent dividend growth, consistent with the differing cyclicality of each type's optimism about the risky asset. The second is that the extrapolators' shadow riskfree rate is less than the sophisticates' over the majority of the $\hat{g}$ range. Even when extrapolators have correct beliefs about next period's dividend growth distribution (at $\hat{g} = \bar{g}$), they have a much greater demand for the riskfree asset. This is due to hedging concerns. Because extrapolators believe risky asset returns exhibit positive serial correlation, they believe that risky asset returns are positively correlated with future investment opportunities. In contrast, sophisticates know that stocks are a good hedge, which increases their willingness to hold them. Finally, the extrapolator shadow riskfree rate is less volatile than the sophisticate rate. This is partly because, as we will see in Section 4.2, extrapolators' expected consumption growth does not vary as much as sophisticates'.

Figure 3 shows the price-dividend ratio at the 50% extrapolator wealth cross-section. Extrapolators' procyclical optimism about stocks generates a procyclical price-dividend ratio. Because the price-dividend ratio is stationary and the dividend growth process is independently and identically distributed, procyclical price-dividend ratios must lead to countercyclical expected stock returns in the absence of bubbles. Table 3 shows the results of regressions predicting excess log returns over the next 1, 2, and 4 years using the log price-dividend ratio. Consistent with the regressions using the historical data, the R^2 and coefficient on the log price-dividend ratio in the median simulation run increase with horizon. The median log price-dividend ratio coefficient is more negative and the median R^2 smaller than the corresponding historical estimates. However, the empirical values fall comfortably within the middle 90 percentile range of simulation outcomes.

Figure 4 plots the expected stock return and volatility at the 50% extrapolator wealth cross-section under the true and extrapolators' probability measures. Extrapolators' return beliefs move in the wrong direction in response to realized dividend growth. On the other hand, their procyclical volatility expectations are very accurate. The procycliality of stock volatility is consistent with the findings of Campbell (1987) and Glosten, Jagannathan, and Runkle (1993) that stock volatility is negatively related to expected returns.[13]

4.2 Investor actions

Figure 5 shows the consumption-wealth ratio of sophisticates and extrapolators at the 50% extrapolator wealth cross-section. Because investors have an EIS greater than unity, higher expected returns lead to a lower consumption-wealth ratio. Hence, sophisticates have procyclical consumption-wealth ratios and extrapolators have countercyclical ratios. In addition, extrapolators' consumption-wealth ratio is everywhere above the sophisticates'. This is because extrapolators believe stocks have poor hedging properties, and hence they perceive the investment opportunity set to be less attractive than sophisticates do. Deteriorations in the investment opportunity set will decrease the savings rate when the EIS is greater than unity. This result suggests a new mechanism that might explain some of the negative correlation between financial sophistication and savings rates, a phenomenon frequently attributed to differential self-control or foresight.

Figure 6 shows the first and second moments of consumption growth for investors who survive into the next period, along with the beliefs extrapolators have about these moments for themselves. Consistent with the consumption-wealth ratio graph, sophisticate expected consumption growth is decreasing in $\hat{g}$ but extrapolator expected consumption growth is increasing. Extrapolators believe that the relationship between their consumption growth and $\hat{g}$ is steeper than it actually is. This is becauuse when $\hat{g}$ is above its mean, extrapolators are not as rich next period as they had expected, whereas the reverse is true when $\hat{g}$ is below its mean.

Although extrapolators are consistently mistaken about their conditional mean consumption growth, they have very accurate beliefs about their consumption growth volatility. Sophisticate consumption growth volatility is strongly procyclical, whereas extrapolator volatility is mildly countercyclical. Furthermore, the annualized standard deviation of log sophisticate consumption growth is substantially higher than that of extrapolators: 11.8% versus 1.8% in the median simulation run. The sophisticate consumption growth volatility is not far

[13] On the other hand, Bollerslev, Engle, and Wooldridge (1988) and French, Schwert, and Stambaugh (1987) find a weak positive correlation between stock volatility and returns. Also, the model is not consistent with the negative correlation between stock return innovations and volatility innovations in empirical data.

from the 15.8% average standard deviation Aït-Sahalia, Parker, and Yogo (2003) estimate for the extremely wealthy, who are likely to be disproportionately sophisticated.

5 Conclusion

This paper has argued that the equity premium and volatility puzzles arise from the presence of extrapolative investors who believe that high past stock market returns predict high future returns. When the elasticity of intertemporal substitution is greater than unity, this belief causes extrapolator savings rates to vary procyclically, pushing up price-dividend ratios during consumption booms and pushing down price-dividend ratios during consumption busts. The resulting predictability of stock returns causes sophisticated investors with rational expectations to make their savings rates countercyclical, increasing the covariance between their consumption growth and stock returns. Sophisticates therefore demand a high unconditional equity premium because of this high covariance. Extrapolators do not have high consumption growth covariance with stock returns, but they demand a high unconditional equity premium because they believe stocks are a poor hedge due to their perceived positive return serial correlation. Aggregate consumption statistics include both extrapolator and sophisticate consumption, hiding from the econometrician the stock market's consumption risk to sophisticates.

I numerically solve a general equilibrium model of an endowment economy that is populated by sophisticated investors and extrapolators. I find that I can generate a low, stable riskfree rate and a high equity Sharpe ratio consistent with the postwar U.S. data with a relative risk aversion coefficient of 4 and an elasticity of intertemporal substitution (EIS) of 2 when extrapolators constitute 75% of the population.

Numerous empirical studies have documented that extrapolative beliefs about stock returns are commonly held in the population, even among the rich. Nonetheless, it seems plausible that the rich are more likely to hold rational beliefs than the poor. Consistent with this notion, empirical studies have shown that the covariance of consumption growth with stock returns is higher for the rich. The model in this paper predicts that identifying the subset of investors who understand that stock returns are mean-reverting and measuring their consumption growth will yield an even higher covariance. Furthermore, the unconditional average portfolio share in cash and Treasury bills will be increasing with the strength of a household's extrapolative tendencies, since knowledge of mean reversion in stock returns generates a positive hedging demand for equities.

6 Appendix: Computational algorithm

I solve the model numerically using the time-iteration algorithm described in Judd, Kubler, and Schmedders (2000). I approximate the price-dividend and extrapolator consumption-wealth functions by using a two-dimensional cubic spline with knots at the grid points $\{\omega_k\} \times \{\hat{g}_k\}$ and the "not-a-knot" end condition. There are 21 knots in $\{\omega_k\}$ distributed evenly between 0 and 1 inclusive. I use 7 knots in $\{\hat{g}_k\}$, where there is one knot at $\bar{g}$ and 3 knots spaced evenly in each direction around the center knot, such that a 6-standard-deviation range of $\hat{g}$ is covered. I constrain $\hat{g}_{t+1}$ to remain at the boundary if a dividend realization would take it off the grid.

The time-iteration algorithm continues until the maximum absolute difference between successive policy functions evaluated at any grid point is less than 10^{-5}.

The time-iteration algorithm is not globally convergent; like most numerical algorithms, it requires a starting point that is not too far from the final solution. My initial guesses for the policy functions are the solutions when there are only extrapolators in the economy.

Riskfree rates for the simulations are determined by calculating the riskfree rate at each grid point and using cubic spline interpolation to approximate the riskfree rate for points not on the grid.

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Table 1: Parameter Values

This table gives the parameter values used for the model. Where appropriate, numbers have been annualized, and any calculations used to annualize the numbers are reflected in the second column. For example, in order to annualize mean log consumption dividend growth, the variable $\overline{g}$ has been multiplied by 4, so the corresponding entry in the second column is $4\overline{g}$.

Parameter	Variable	Value
Mean log consumption dividend growth	$4\overline{g}$	1.964%
Standard deviation of log consumption dividend growth	$2\sigma_g$	4.449%
Extrapolator belief persistence coefficient	ϕ^4	0.784
Expected investing lifespan	$1/4\delta$	50 years
Proportion of extrapolators in population	Q	0.75
Elasticity of intertemporal substitution	ψ	2
Coefficient of relative risk aversion	γ	4
Time discount factor	β^4	0.974

Table 2: Simulated Annualized Moments

The second column gives annualized moments from simulating the model for 1,207 quarters and discarding the first 1,000 quarters. The top number in each cell is the median moment from 1,000 simulation runs. The numbers in brackets below are the 5th and 95th percentile moments from the simulation runs. The third column gives the moments from an economy populated only by sophisticated investors. The fourth column gives actual moments from postwar U.S. data, taken from the data in Campbell (2003). To obtain the log equity premium, I calculate the mean of the equity return divided by the riskfree rate, and then I take the log of that mean. The log equity Sharpe ratio is the log equity premium divided by the log equity return standard deviation.

	Model simulation results	Sophisticate-only economy	U.S. data, 1947.2-1998.4
Mean log riskfree rate	0.95%	5.39%	0.90%
	[0.76, 1.07]		
Log riskfree rate	0.16%	0%	1.75%
standard deviation	[0.10, 0.29]		
Log equity premium	3.65%	0.60%	7.19%
	[2.56, 4.82]		
Log equity return	7.98%	4.45%	15.74%
standard deviation	[7.19, 8.70]		
Log equity Sharpe ratio	0.458	0.135	0.457
	[0.339, 0.589]		

Table 3: Predicting Excess Log Returns with the Log Price-Dividend Ratio

This table presents the results of regressions that predict future excess equity returns using the log price-dividend ratio. The dependent variables are the log of the mean of equity returns divided by the riskfree rate over the next 1, 2, and 4 years, scaled to give an average quarterly excess return. The explanatory variables are the log price-dividend ratio and a constant. The first column gives the results from running the regression on each of 1,000 simulation runs of the model. For each simulation run, 1,207 quarters were simulated and the first 1,000 quarters were discarded. The top number in each cell of the second column is the median coefficient from the 1,000 regressions. The numbers in brackets below are the 5th and 95th percentile coefficients. The third column gives the results from running the regression on postwar U.S. data taken from Campbell (2003).

	Model simulation	U.S. data, 1947.2-1998.4
	Panel A: 1-year future returns	
Slope	-0.3682	-0.1709
	[-0.8346, -0.0330]	
R^2	4.7%	9.6%
	[0.2, 15.2]	
	Panel B: 2-year future returns	
Slope	-0.6648	-0.3459
	[-1.4304, -0.0472]	
R^2	8.7%	20.2%
	[0.3, 26.3]	
	Panel C: 4-year future returns	
Slope	-1.1185	-0.5526
	[-2.1896, -0.0825]	
R^2	14.8%	29.9%
	[0.4, 41.2]	

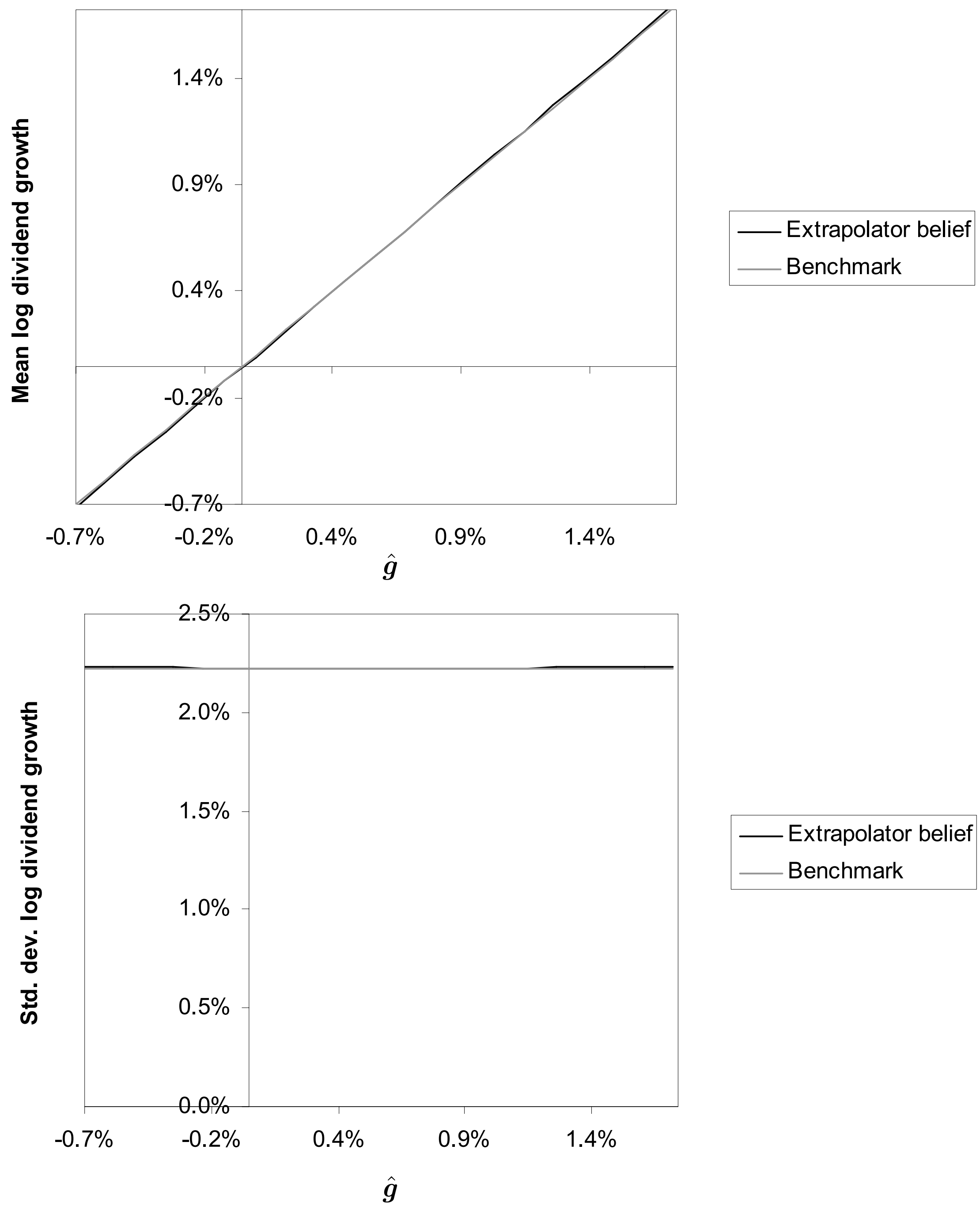


Figure 1: Extrapolator Beliefs About the Moments of Log Dividend Growth. The top panel shows extrapolators' belief about mean log dividend growth in the next period as a function of $\hat{g}$, the weighted average of current and past log dividend realizations. The bottom panel shows extrapolators' belief about the standard deviation of log dividend growth as a function of $\hat{g}$. The grey line in each figure denotes what those beliefs should be if the discrete approximation used to generate these beliefs were perfect.

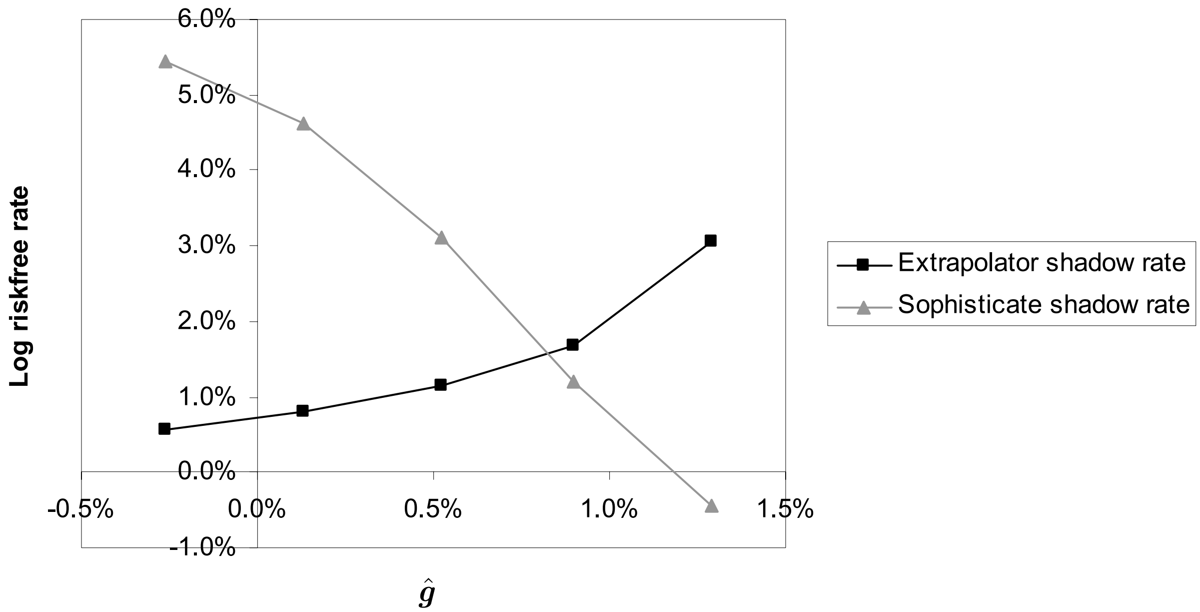


Figure 2. Shadow Annualized Log Riskfree Rates for Each Investor Type. The graph is from the cross-section of the state space where extrapolators hold 50% of the wealth. The distance between adjacent data markers in a series represents one standard deviation of $\hat{g}$, and the middle marker lies at the unconditional mean of $\hat{g}$. The prevailing riskfree rate is the lower envelope of the two series.

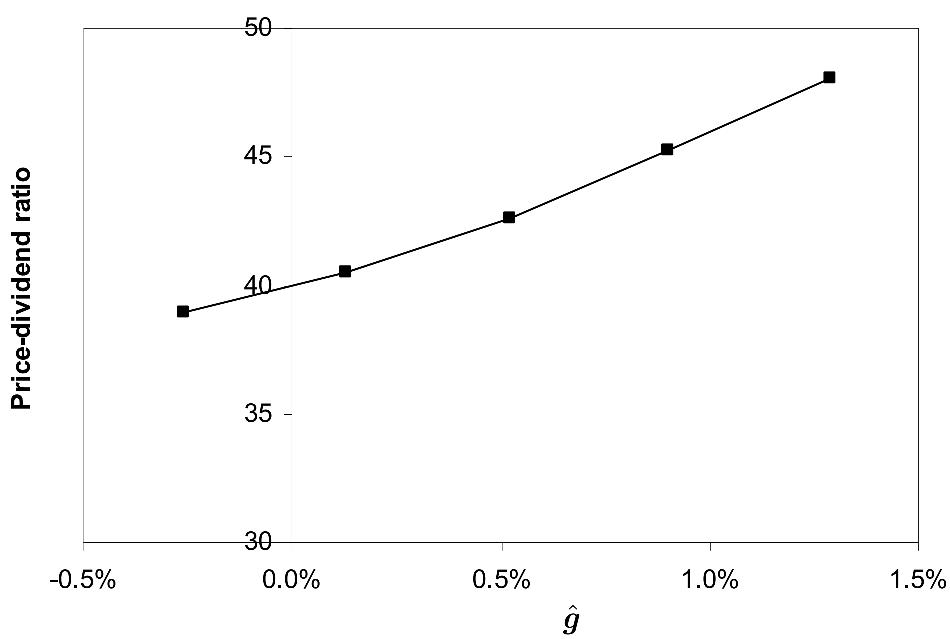


Figure 3. Equity Price-Dividend Ratio. The graph is from the cross-section of the state space where extrapolators hold 50% of the wealth. The dividend in the denominator has been multiplied by 4 to annualize it. The distance between adjacent data markers represents one standard deviation of $\hat{g}$, and the middle marker lies at the unconditional mean of $\hat{g}$.

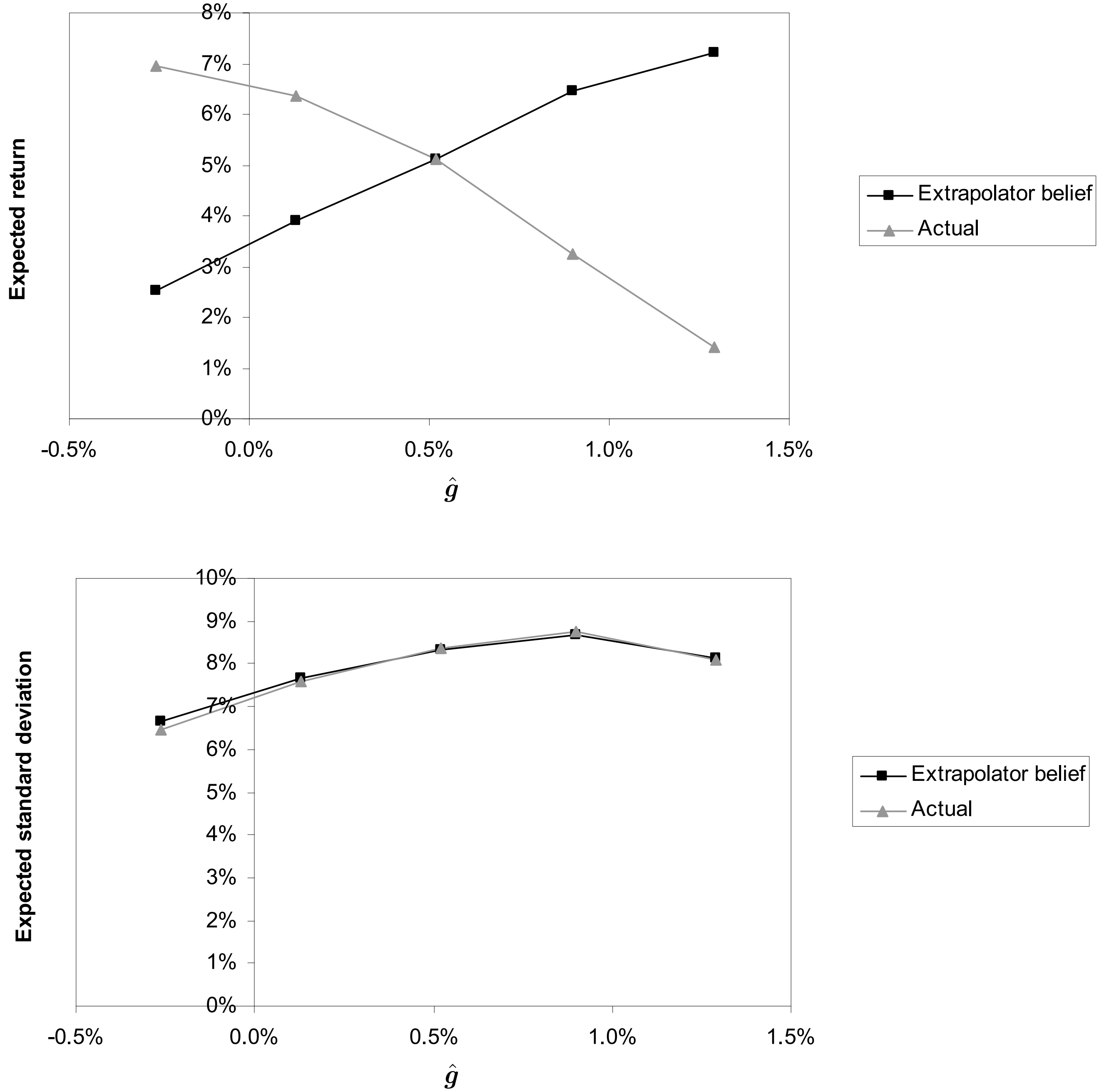


Figure 4. Expected Equity Return and Standard Deviation. The graphs show the actual moments of equity returns and what extrapolators believe them to be at the cross-section of the state space where extrapolators hold 50% of the wealth. Moments have been annualized. The distance between adjacent data markers in a series represents one standard deviation of $\hat{g}$, and the middle marker lies at the unconditional mean of $\hat{g}$.

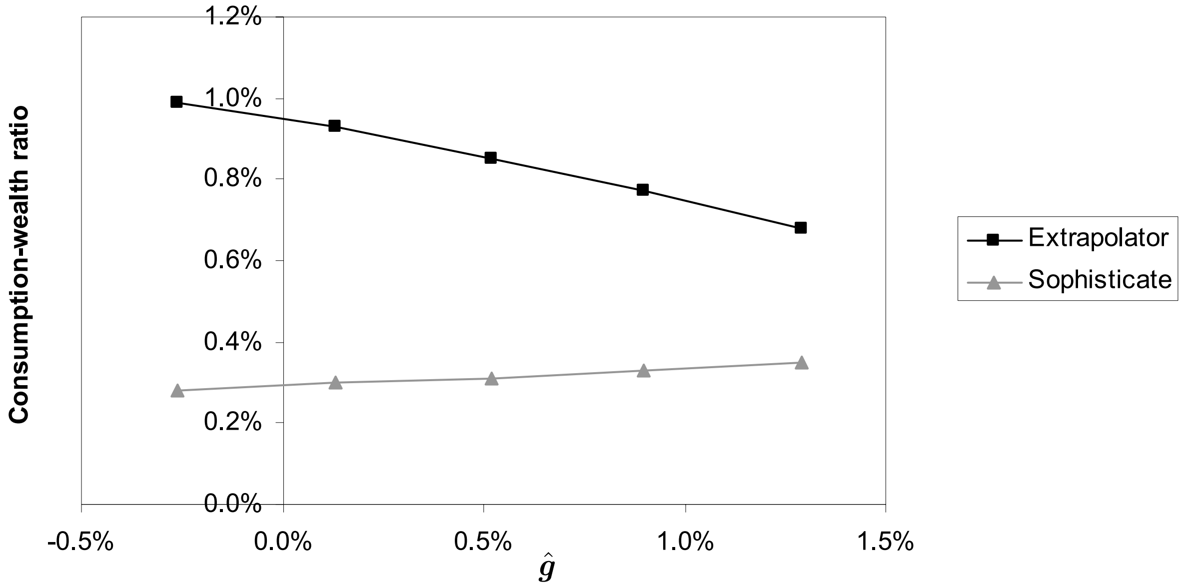
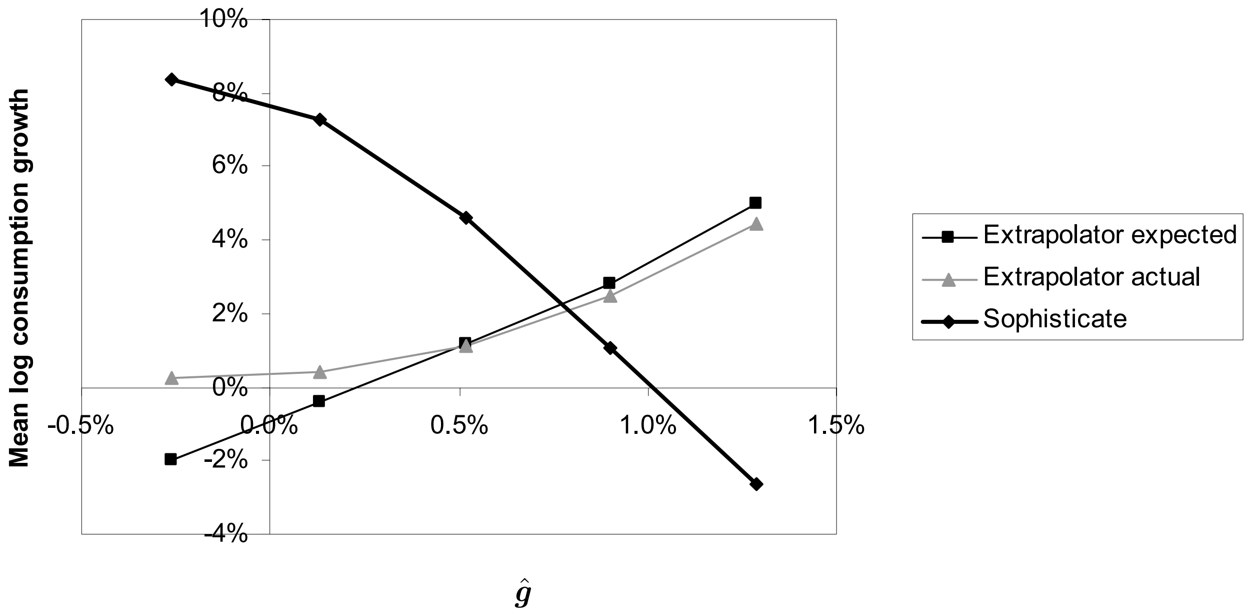


Figure 5. Consumption-Wealth Ratios. The graph shows consumption-wealth ratios of sophisticates and extrapolators at the cross-section of the state space where extrapolators hold 50% of the wealth. The distance between adjacent data markers in a series represents one standard deviation of $\hat{g}$, and the middle marker lies at the unconditional mean of $\hat{g}$.



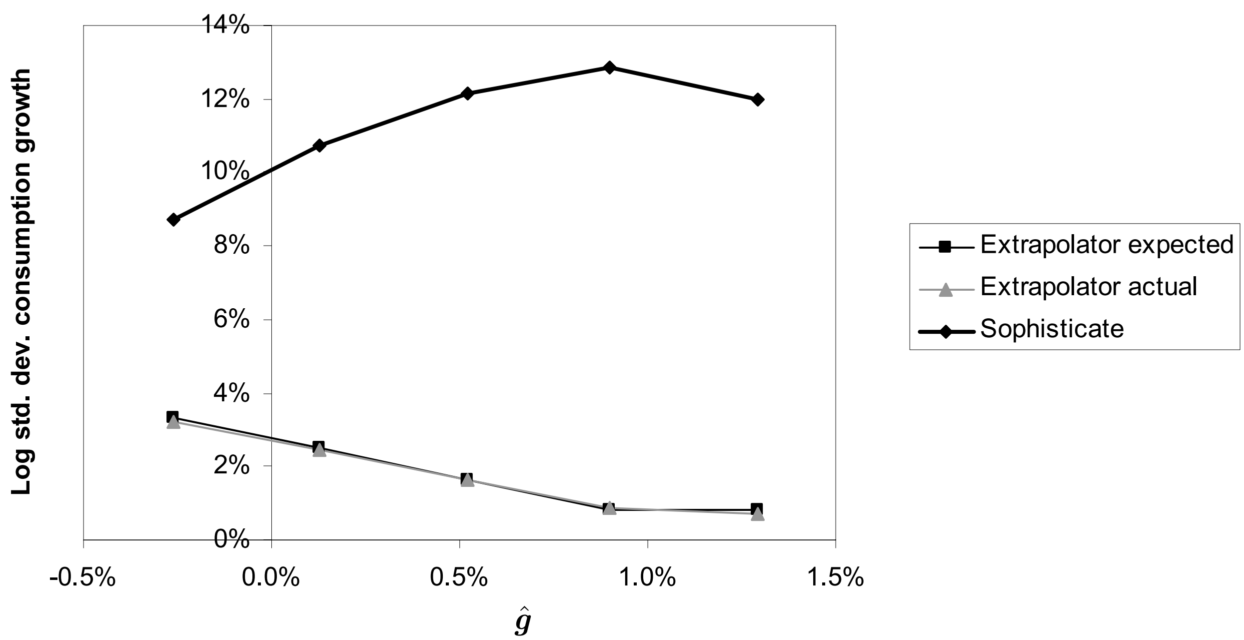


Figure 6. Investor Consumption Growth Moments. These graphs show what extrapolators believe the first two moments of their consumption growth will be, their actual moments, and sophisticates' consumption growth moments. They are taken from the cross-section of the state space where extrapolators hold 50% of the wealth. Consumption growth is measured only for investors who are alive in both the current period and the subsequent period. The distance between adjacent data markers in a series represents one standard deviation of $\hat{g}$, and the middle marker lies at the unconditional mean of $\hat{g}$.